Amplification Matrix Activity
Cosmology Crash Course

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The magnification matrix $\mathcal{M}$ gives the linear transformation from the source plane to the image plane:

$$\vec{\theta} = \mathcal{M}\vec{\beta}$$

It is given by

$$\mathcal{M} = \frac{1}{(1-\kappa)^2 - (\gamma_1^2 + \gamma_2^2)} \begin{pmatrix} 1-\kappa+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa-\gamma_1 \end{pmatrix}$$

where κ, γ_1, and γ_2 are constants determined by the lensing mass distribution and redshift. We'll see where they come from later; for now let's get a feel for what they do.

Open the website at: http://www.ies.co.jp/math/java/misc/don_trans/don_trans.html and scroll down to the applet.

Find the magnification matrix in the following cases, and see how the image changes.

A) $\kappa = 1/6, \gamma_1 = \gamma_2 = 0$

B) $\kappa = 0, \gamma_1 = 0, \gamma_2 = 1/6$

C) $\kappa = 0, \gamma_1 = 1/6, \gamma_2 = 0$

Try different combinations of the above, and try several of your own values for κ, γ_1, and γ_2. Get a feel for what they do the image.

D) We call γ_1 and γ_2 the "shear". (Sometimes we just use $\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$.) Why do you think this is called the shear?

E) We call κ the "convergence". Any ideas why this name might have been given?

F) If given θ (what we actually observe), how would you find β?

G) (Bonus, if time) Is the Magnification Matrix symmetric? What does this tell you about its eigenvectors? Writing $\gamma_1 \equiv \gamma \cos 2\phi$ and $\gamma_2 \equiv \gamma \sin 2\phi$, find the eigenvectors in terms of κ and γ. Find the determinant. What does this tell you about the transformation?