Solving the Lens Equation for a Point Mass
Cosmology Crash Course

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From general relativity, the deflection angle $\hat{\alpha}$ is given by

$$\hat{\alpha} = \frac{4GM}{c^2\xi}$$

for a point mass of mass M.

A) What would the defection angle be if the point mass were replaced by a spherical mass of mass M and radius $R < \xi$? Explain.

B) Write α in terms of $\hat{\alpha}$, D_{LS}, and D_S. (Remember the distances are angular diameter distances.)

C) Substitute this into the lens equation $\theta = \alpha + \beta$ and rearrange to get a quadratic equation in θ.

D) Note that this quadratic equation can be simplified by defining:

$$\theta_E = \sqrt{\frac{D_{LS}}{D_L D_S}\frac{4GM}{c^2}}$$

E) Solve the quadratic equation. What are the locations of the two images relative to the position of the lens? (Are the above the lens or below the lens, as drawn in the diagram on the board?)

F) (Bonus) Solve the lens equation when the lens is uniform sheet with a constant surface mass density Σ_o.